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A Dynamic Model for C^3 Information
Incorporating the Effects of Counter C^3

by

Paul H. Moose

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A Dynamic Model for C^3 Information Incorporating the Effects of Counter-C^3

Paul H. Moose

I. Introduction

In modern warfare, an operational commander is intimately concerned with the quality, timeliness and completeness of his "picture" of the tactical situation. To a very large extent, his fortunes and those of his assigned forces depend on his having available, when and where he needs it, accurate data about the status, location and activities of both his own and the enemies' forces. Similar requirements extend well down into subordinate echelons of his command, including individual unit commanders and even "smart" weapons.

The methods by which necessary information is acquired are remarkably diverse. Included, in our "computer age", are sophisticated radar and intercept equipments, a variety of imaging systems and acoustic sensors as well as ordinary direct reports and observations from the commander's own personnel. Inputs from special intelligence channels, and many other categories of reports constantly arriving by a variety of means round out a massive and continuing informational input. The staff, assisted undoubtedly by modern automatic data processing equipments, is regularly creating and updating their assessment of the situation in order to give the best operational picture they can to their commander. The commander will, to a very great degree, make rational and reasonably predictable decisions for the future activities of his forces based on the world view he has developed from this sequence of images.

II. A View of Information

We wish at this point to make several important observations about this "image of reality" that the commander works with. First, the images he has are never absolutely correct, that is, they contain errors. Nor are they perfectly sharp, that is, there are always many questions that are unanswered, or elements of contradiction or ambiguity. Secondly, an image gets fuzzier and fuzzier the further into the future one attempts to extrapolate it. This is because most elements of the picture are dynamic, i.e. they change (location, behavior, etc.) with time. The attributes of the elements may be partially constrained. For example, a ship cannot move faster than about 30 knots. However, after sufficient time most features of the picture will have totally relaxed, and may have taken on any of their possible values or conditions.

This "fuzzyness in the crystal ball" axiom has a corollary. If the commander loses, or turns off, his senses or sources of information, his "current image" will grow fuzzier and fuzzier with time until it is completely blurred. Put another way, a commander only maintains his uncertainty about what is going on below its worst possible level by virtue of the continual application of systemic resources to guarantee an inflow of new information.

The second law of thermodynamics holds that entropy(disorder) will grow to its maximum possible value within the constraints of the system. We shall identify uncertainty with entropy. Thus sensory devices and information sources provides the constraints on uncertainty. They do this by continually importing information (negentropy) to offset uncertainty's inevitable growth.

III. Birth and Death of Uncertainty

We postulate that if left unconstrained, uncertainty will grow from its current value toward its maximum possible value H_{max} with a rate proportional to the remaining knowledge. (H_{max} measures the worst case, total ignorance, where all possibilities are equally probable.)[1] If T_s is the systemic relaxation time, then

$$\dot{H}_B = T_s^{-1} \quad (H_{MAX} - H) \qquad (1)$$

models the birth of uncertainty, $\dot{H}_B$, as proportional to the remaining knowledge, H_{MAX} -H, divided by the system relaxation time T_s. Left unconstrained, uncertainty will grow as shown in Figure 1.

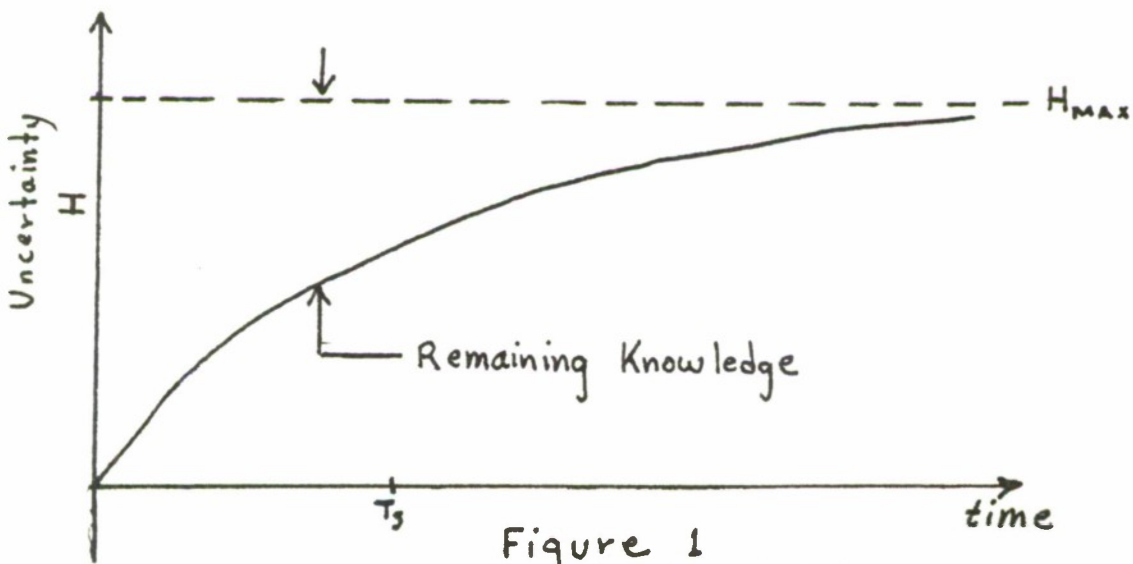


Figure 1
Growth of Uncertainty

[1] Growth of this type is not an unreasonable assumption as can be seen in the work of Moose and Harrison, "An Analytic Model of Coordinated Effort with Application to Surveillance C^3", May 1979, AD # A071-081.

However, the commander is continually receiving new data that helps to reduce uncertainty; he is receiving "negentropy". We assume that the rate of entropy reduction is directly proportional to his state of ignorance. Thus

$$\mathring{H}_D = -T_I^{-1} \; H \tag{2}$$

expresses the fact that arriving information destroys uncertainty at a rate in direct proportion to its current value. T_I is a characteristic negentropy arrival time. Let us further assume that the causes (1) and (2) may be superimposed such that

$$\mathring{H} = T_S^{-1} \; H_{MAX} \; -(T_S^{-1} + T_I^{-1}) \; H \tag{3}$$

models the net growth (or reduction) in uncertainty at any point in time. It is clear that this system is stationary when

$$\frac{H}{H_{MAX}} = \frac{T_I}{T_S + T_I} \tag{4}$$

since at this point the birth and death rates exactly cancel one another and $\mathring{H} = 0$.

Although this simple first order linear system is not very sophisticated, we see that it already illustrates some practical features. When T_S, the systemic relaxation time is very short compared to the characteristic arrival time of new information, uncertainty finds its equilibrium near maximum. On the other hand, if the relaxation time is equal to the characteristic arrival times, uncertainty will be cut to one-half its maximum value. When information arrives at a much greater rate than the attributes of the picture can change, uncertainty is very low and finds equilibrium at a level in direct proportion to the ratio T_I/T_S. One must be cautious and take due note of the fact that T_I & T_S (or $V_I = T_I^{-1}$ and $V_S = T_S^{-1}$) are global or macroscopic system variables. V_I & V_S might have

units, for example, of bits/bit per hour and refer to the average behavior of the entire system ensemble much as species birth and death rates are typically measured in births (or deaths)/unit of population per year, and describe an average of the entire population.

IV. Information War

Rona[2] has described the concept of "information war" as a dominant factor in the conduct of modern warfare. In an information war, one actively attempts to deny the enemy knowledge of his force positions, numbers, intentions, etc. This is done by a variety of means. Included, for example, are cover and deception tactics, distribution of radar chaff, decoys, false messages, etc. One also works to keep his own communications intact and secure, but intercepts and exploits and/or jams those of the enemy. One may also try to physically disable enemy C^3 facilities and channels. In all of this, the purpose is to try to reduce one's own uncertainty by assuring a steady, reliable inflow of relevant information, a term we have already described above. But moreover, to disrupt the opposition's flow of information and ultimately blur or distort his image of the operational situation. This will be cause, we maintain, for poor decisions on his part thereby enhancing one's own force effectiveness.

Let us suppose X & Y represent the entropies of two opposing sides. We should include, in $\overset{\circ}{X}$, a "counter-C^3" term to represent informational disruption by Y, and vice versa. Thus, the rate equations take the form

$$\begin{aligned} \overset{\circ}{X} &= V_{SX} X_{MAX} - (V_{SX} + V_{IX})X + \Gamma_{YX}(Y_{MAX} - Y) \\ \overset{\circ}{Y} &= V_{SY} Y_{MAX} - (V_{SY} + V_{IY})Y + \Gamma_{XY}(X_{MAX} - X) . \end{aligned} \tag{4}$$

---

(2) Rona, T.P., "Weapon Systems and Information War", Boeing Aerospace Co., Seattle, WA, July 1976.

We have assumed that Y's disruption of X's flow of information is directly proportional to his current knowledge, $Y_{MAX}-Y$, and likewise for X's disruption of Y. The "Counter-C^3 coefficients", Γ_{YX} & Γ_{XY} are, we presume, positive if in fact the information war is having the desired results, at least on the average.

There is a question, however, about how these coefficients are to be chosen. In particular does Γ_{YX} depend only on Y's efforts against X or is it also dependent on X's own knowledge, and if so, how? It seems plausible to suppose that if X is very short of knowledge of the situation already, he may be difficult to confuse even more, whereas if his knowledge is great he may be much more susceptible to disruption, deception, decoys and jamming, Admittedly, this is a highly speculative argument, but is is an extremely important point because it determines whether the two systems are linearly or non-linearly coupled, which, as we shall see presently, has an immense influence on their dynamic behavior.

Let us list some options for Y's coupling to X.

Γ_{YX}	Form	Result
a) $\Gamma_{YX} = \gamma_{YX}X_{MAX} \geqslant 0$	constant	linear coupling
b) $\Gamma_{YX} = \gamma_{YX}(X_{MAX} - X) \geqslant 0$	proportional to X's knowledge	2nd degree (non-linear) coupling
c) $\Gamma_{YX} = \gamma_{YX}X \geqslant 0$	proportional to X's uncertainty	2nd degree (non-linear) coupling
d) $\Gamma_{YX} = \gamma_{YX}X_{MAX}\, e^{-X} \geqslant 0$	increases exponentially with X's knowledge	non-linear coupling
e) $\Gamma_{YX} = \gamma_{YX}\ X(X_{max} - X) \geqslant 0$	maximum coupling when $X = \frac{X_{MAX}}{2}$ and no coupling when $X = 0$ or $X = X_{MAX}$	3rd degree (non-linear) coupling

If we chose case b) above as the most intuitively appealing dependence, then Eq's (4) take the form

$$\dot{X} = (V_{SX} + \gamma_{YX} Y_{MAX})X_{MAX} - (V_{SX} + V_{IX} + \gamma_{YX} Y_{MAX}) X - \gamma_{YX} Y(X_{MAX} - X)$$

$$\dot{Y} = (V_{SY} + \gamma_{XY} X_{MAX})Y_{MAX} - (V_{SY} + V_{IY} + \gamma_{XY}X_{MAX})Y - \gamma_{XY}X(Y_{MAX} - Y) \tag{5}$$

which are 1st order, 2nd degree - non-linear coupled differential equations. We may write them more compactly as,

$$\dot{X} = \alpha_o X_{MAX} - \alpha_1 X - \alpha_2 Y(X_{MAX} - X)$$

$$\dot{Y} = \beta_o Y_{MAX} - \beta_1 Y - \beta_2 X(Y_{MAX} - Y) \tag{6}$$

To summarize, we have the following relationships between coefficients and definitions of system parameters:

A. Coefficient Relationships

i) $\alpha_o = (V_{SX} + \alpha_2 Y_{MAX})$, $\beta_o = (V_{SY} + \beta_2 X_{MAX})$

ii) $\alpha_1 = \alpha_o + V_{IX}$, $\beta_1 = \beta_o + V_{IY}$

iii) $\alpha_2 = \gamma_{YX}$ $\beta_2 = \gamma_{XY}$

B. Parameter Definitions

i) X_{MAX}, Y_{MAX} ; Maximum Uncertainty (Bits)

ii) V_{SX}, V_{SY} ; Uncertainty Birth Rates (bits/bit per unit time)

iii) V_{IX}, V_{IY} ; Uncertainty Death Rates resulting from data inputs (bits/bit per unit time)

iv) γ_{YX}, γ_{XY} ; Counter C^3 coefficients (bits^{-1} per unit time)

Note that if the counter C^3 coefficients are both zero, Eq's (6) revert to a pair of uncoupled equations of the form given by Eq. (3).

V. Analysis

Analysis of the behavior of the non-linearly coupled 1st order differential equations, presented in Eq's (6) as a model for information dynamics between opposing C^3 systems, is usefully subdivided into separate treatments of 1.) "Stationary" or "Equilibrium" conditions, 2.) dynamic behavior near equilibrium points, and 3.) dynamic behavior far from equilibrium points. Before proceeding to the discussion of each of these, it is convenient to consider a normalized version of the equations as follows. Let

$$
\begin{aligned}
\dot{x} &= [(v_{sx} + \gamma_{yx}) - (v_{sx} + v_{ix} + \gamma_{yx})\, x - \gamma_{yx}\, y(1-x)]\, Y_{max} \\
\dot{y} &= [(v_{sy} + \gamma_{xy}) - (v_{sy} + v_{iy} + \gamma_{xy})\, y - \gamma_{xy}\, x(1-y)]\, X_{max}
\end{aligned}
\tag{7}
$$

where we now have defined normalized entropy variables and entropy rates,

$$x = X/X_{MAX},\ \dot{x} = \dot{X}/X_{MAX}\ ,\ \ y = Y/Y_{MAX},\ \text{and}\ \dot{y} = \dot{Y}/Y_{MAX}.$$

Note that $0 \leq x, y \leq 1$ for physically realizable conditions. We may again introduce the more compact form

$$
\begin{aligned}
\dot{x} &= (\alpha_o - \alpha_1 x - \alpha_2\, y(1-x))\, Y_{max} \\
\dot{y} &= (\beta_o - \beta_1 y - \beta_2 x(1-y))\, X_{max}
\end{aligned}
\tag{8}
$$

with the coefficients defined now as follows:

A. Normalized Coefficients

i) $\alpha_o = v_{sx} + \gamma_{yx}$, $\beta_o = v_{sy} + \gamma_{xy}$

ii) $\alpha_1 = \alpha_o + v_{ix}$, $\beta_1 = \beta_o + v_{iy}$

iii) $\alpha_2 = \gamma_{yx}$, $\beta_2 = \gamma_{xy}$

B. <u>Normalized Parameters</u>

i) $v_{sx} = V_{SX}/Y_{MAX}$, $v_{sy} = V_{SY}/X_{MAX}$; Normalized Birth Rates (bits^{-1}/unit time)

ii) $v_{ix} = V_{IX}/Y_{MAX}$, $v_{IY} = V_{IY}/X_{MAX}$; Normalized Death Rates (bits^{-1}/unit time)

iii) γ_{xy} , γ_{yx} ; Counter C^3 coefficients (bits^{-1}/unit time) the same as in Eq's (6)).

V.1 <u>Stationary Points & Sensitivity</u>

Stationary points will occur when $\dot{x} = \dot{y} = 0$, that is when the conditions

$$
\begin{aligned}
\alpha_o - \alpha_1 x - \alpha_2 y(1-x) &= 0 \\
\beta_o - \beta_1 y - \beta_2 x(1-y) &= 0
\end{aligned}
\qquad (9)
$$

are met by x and y.

Eq (9) may be solved for the values of x and y that yield these conditions. Because of the presence of the product term xy, there are, in general, two stationary points, which we shall designate (x_1, y_1) and (x_2, y_2). Before we look at the general solutions of (9), consider the case where $\gamma_{xy} = \beta_2 = 0$ and $\gamma_{yx} = \alpha_2 > 0$. This case, which we shall designate as "Case 1", models the conditions where X is exercising no counter-C^3 on Y, but Y is actively counter-C^3ing X. We wish to see what shifts occur in the equilibrium conditions (x_o, y_o) as a function of Y's counter-C^3 effort γ_{yx}.

Observe that the equations are no longer quadratic and there is only one equilibrium point (x_1, y_1) and since $\beta_2 = 0$, $y_1 = y_o$ so that immediately we find

$$
x_1 = \frac{\alpha_o}{\alpha_1} \cdot \frac{\left(1 - \frac{\alpha_2}{\alpha_o}\, y_o\right)}{\left(1 - \frac{\alpha_2}{\alpha_1}\, y_o\right)} \qquad (10)
$$

Thus we see that $Y^{'s}$ state of knowledge is left unchanged but $X^{'s}$ state deteriorates from x_o to x_1, $(x_1 \geqslant x_o)$, as a function of $Y^{'s}$ countering effort $\alpha_2 = \gamma_{yx}$ in accordance with Eq (10). Figure 2 shows a plot of $X^{'s}$ relative increase in entropy, $(x_1 - x_o)/x_o$, versus γ_{yx}/v_{sx}, $Y^{'s}$ countering efforts in relation to $X^{'s}$ natural entropic birth rate. Two curves are shown; the lower is for the case $x_o = 0.1$, $y_o = 2/3$ and the upper is for the case $x_o = 0.1$, $y_o = 0.1$. Note they both approach the value 9 for large values of γ_{yx}. This is maximum entropy, $x_1 = 1$, since we started at $x_o = .1$; i.e. the most Y can do is increase $X^{'s}$ entropy by 9 times since that represents total uncertainty for X. Note for small values in Figure 2 that the curves approach straight 45^o lines on log-log scales indicating that $(x_1 - x_o)/x_o$ and γ_{yx}/v_{sx} are linearly related for small values γ_{yx}. Indeed careful expansion of Eq (10), retaining all first order terms in α_2, $(\alpha_2 = \gamma_{yx})$, shows that for small γ_{yx},

$$(x_1 - x_o)/x_o = \frac{\gamma_{yx}}{v_{sx}} \cdot (1-x_o)(1-y_o). \tag{11}$$

Readers may easily verify for themselves that Eq (11) gives the same results as are shown in Figure 2 for values of $\gamma_{yx}/v_{sx} < 1$.

Eq (11) gives the "counter-C^3 sensitivity" of the system for one way coupling. It shows that if either X or Y are in an initial state of great uncertainty, counter-C^3 efforts of one against the other will be of little value since either $(1-x_o)$ or $(1-y_o)$, or both, are approximately zero. However, if both sides are functioning with high informational efficiency each side is maximally vulnerable to countering efforts by the other.

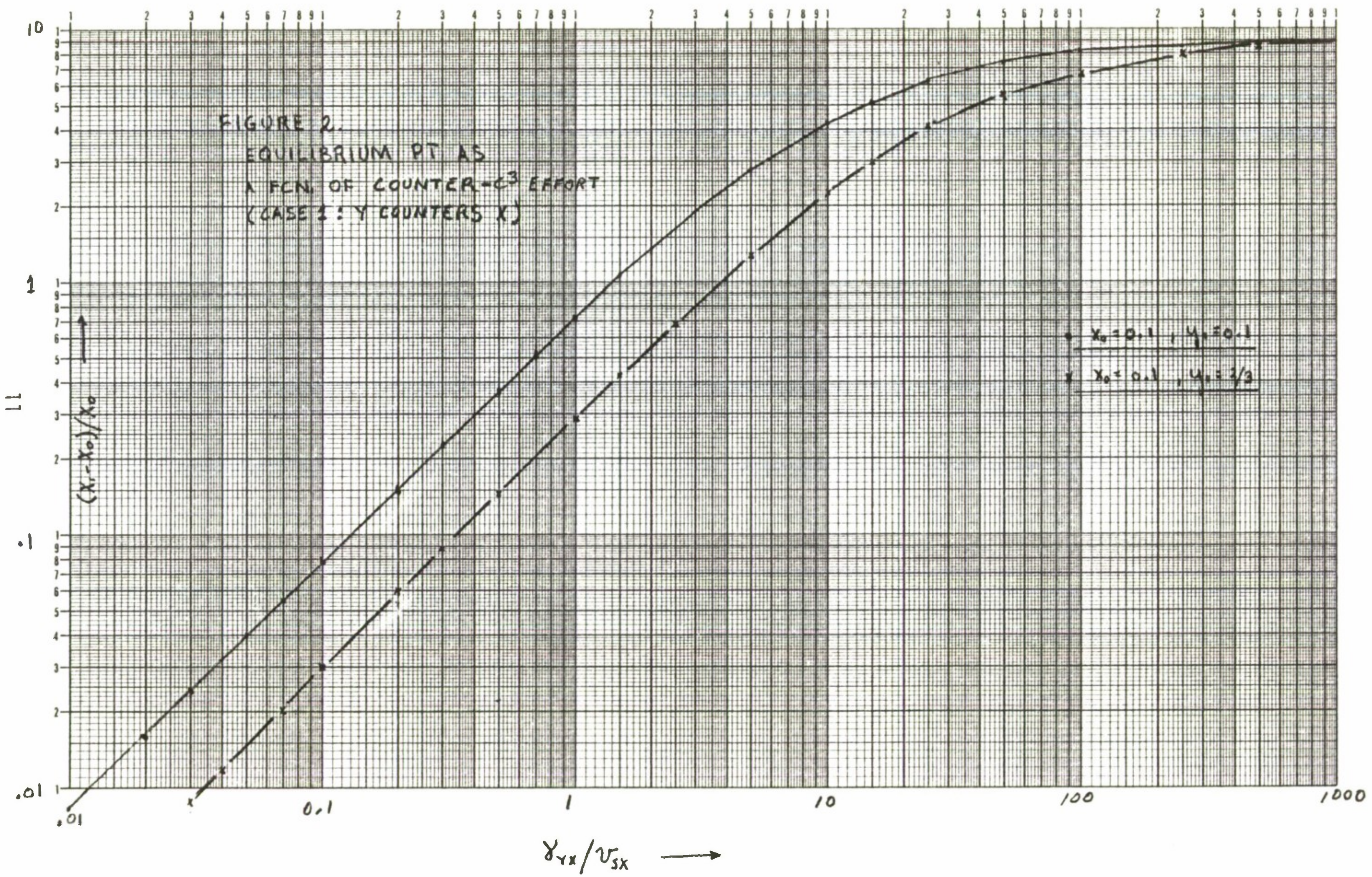


FIGURE 2.
EQUILIBRIUM PT AS
A FCN. OF COUNTER-C3 EFFORT
(CASE 1: Y COUNTERS X)

Having obtained an initial feeling for the sensitivity of the equilibrium point to the coupling factors, let us return to the general problem posed by Eq's (9) where both sides are actively engaged in counter-C^3 efforts. Solving for the roots of Eq's (9), we find

$$y_{1,2} = c_1 x_{1,2} + c_o \qquad (12)$$
$$x_{1,2} = p/2 \pm \frac{1}{2} (p^2 - 4q)^{\frac{1}{2}}$$

where

$$p = 1 + \frac{\alpha_1}{\alpha_2 c_1} - \frac{c_o}{c_1} \qquad (13)$$

$$q = \frac{\alpha_o}{\alpha_2 c_1} - \frac{c_o}{c_1} \qquad (14)$$

$$c_o = (\alpha_2 \beta_o - \alpha_o \beta_2)/(\alpha_2(\beta_1 - \beta_2)) \qquad (15)$$

$$c_1 = (\alpha_1 \beta_2 - \alpha_2 \beta_2)/(\alpha_2(\beta_1 - \beta_2)) \qquad (16)$$

It can be shown that $p^2 - 4q \geq 0$ and therefore the roots (x_1, y_1) and (x_2, y_2) are always real. Further general analysis of Eq's (12) has shown that one of the roots is always found in the physically realizable space $(0 < x_1 < 1, 0 < y_1 < 1)$ and the other is found outside the space, but in the first quadrant.(3)

Considerable practical insight can be obtained by considering the following "matched case" which we shall designate "Case 2". For Case 2, we let

$$\frac{\gamma_{yx}}{\gamma_{xy}} = \frac{\alpha_2}{\beta_2} = \frac{v_{sx}}{v_{sy}} \qquad (17)$$

---

(3) K.E. Woehler, "Root Locations", private communication, 1980.

that is we keep the counter-C^3 efforts in a constant ratio equal to the ratio of natural entropic birth rates. With this constraint, we find that

$$c_o = 0 \quad , \quad c_1 = y_o/x_o \tag{18}$$

so that

$$y_{1,2} = (y_o/x_o) \ x_{1,2}. \tag{19}$$

For Case 2, we see that both the equilibrium points must always lie on a straight line in the x,y plane passing thru the origin and the uncoupled stable point x_o,y_o. Also, we note that

$$p = 1 + \frac{x_o}{y_o} \frac{\alpha_1}{\alpha_2} > 0 \text{ and } q = \frac{x_o}{y_o} \frac{\alpha_o}{\alpha_2} > 0 \tag{20}$$

and therefore all roots are positive. In Figure 3 we have plotted the relative locations of the roots, $(x_{1,2} - x_o)/x_o$, versus γ_{yx}/v_{sx}, the normalized counter-C^3 coefficient for the initial conditions $x_o = y_o = 0.1$. Note that one root pair starts at (∞, ∞) and moves towards $(1,1)$ as γ_{yx} increases whereas the other root pair begins at (x_o, y_o) and moves toward $(1,1)$ with increasing γ_{yx} (and increasing γ_{xy} according to Eq (17)).

By a very careful expansion of the radical $(p^2 - 4q)^{1/2}$ with the additional constraint that $c_1=1$, i.e. that $x_o=y_o$ (met by the initial conditions used in Fig. 3), we find that for small γ_{yx};

$$\frac{x_1 - x_o}{x_o} \cong \frac{\gamma_{yx}}{v_{sx}} (1 - x_o)(1 - y_o) \tag{21}$$

and

$$\frac{x_2 - x_o}{x_o} \cong \frac{1}{(\gamma_{yx}/v_{sx}) \ x_o y_o} \tag{22}$$

for the smaller and larger stationary points respectively. Eq. (21), giving the sensitivity to coupling for the two-way matched case (Case 2), is seen to be identical to the sensitivity for Case 1, the one-way coupling condition

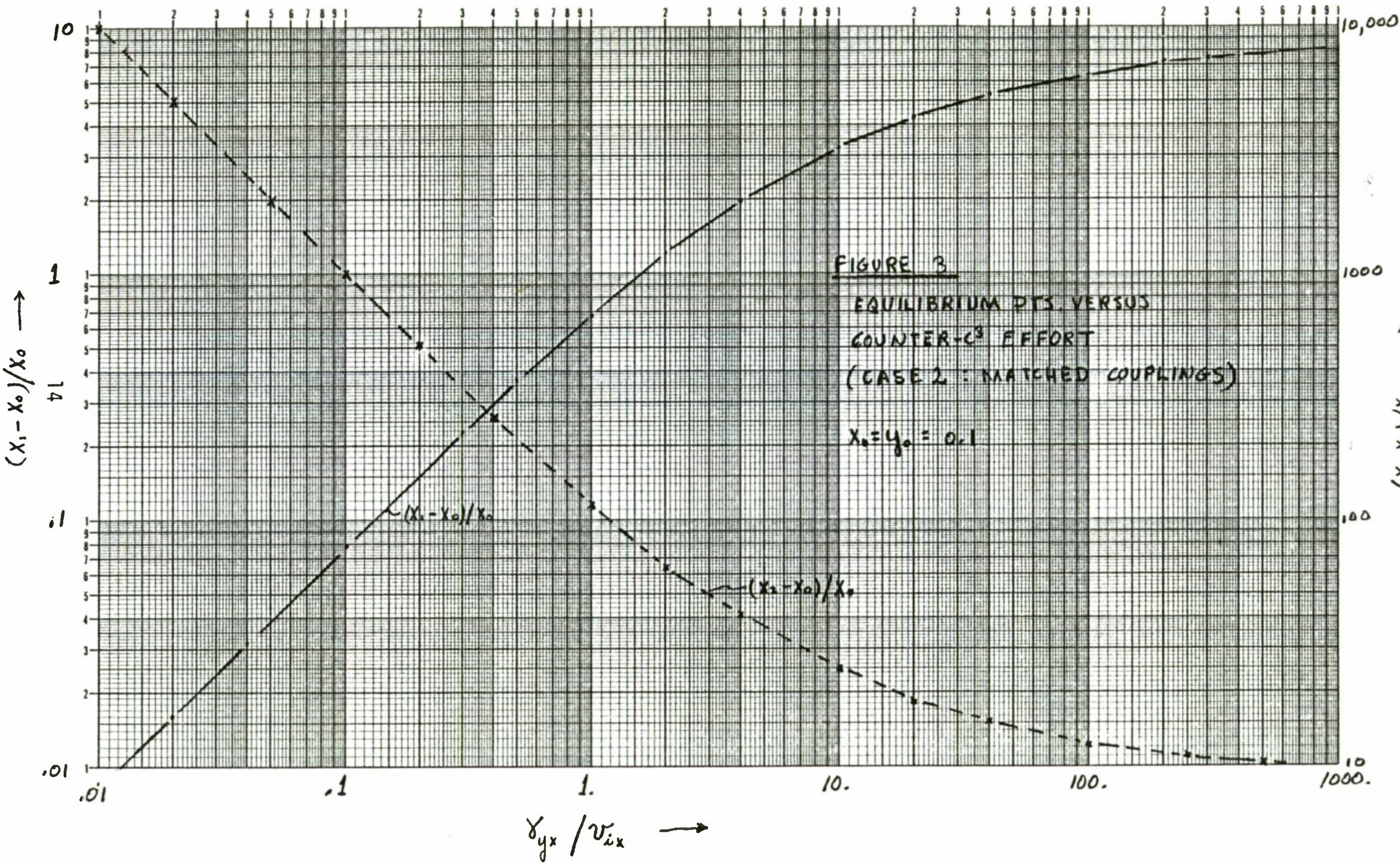


FIGURE 3
EQUILIBRIUM PTS. VERSUS
COUNTER-C³ EFFORT
(CASE 2 : MATCHED COUPLINGS)
X₀ = y₀ = 0.1
(X₁ - X₀)/X₀
(X₂ - X₀)/X₀
γ_yx / v_ix
10
1
.1
.01
.01
.1
1.
10.
100.
1000.
10,000
1000
100
10

given in Eq (11). Eq (22), shows that the second stability point is always very large, i.e. much larger than one and therefore lies outside the physically reliazable uncertainty space.

The important thing to conclude from this analysis is that for small counter-C^3 coefficients, the relative loss of knowledge is proportional to the ratio of counter-C^3 effort to the target's natural entropic birth rate. If the target's information decay is very slow under normal conditions, then he will be very susceptible to counter-C^3 activities. However, if the normal environmental relaxation times, T_s, are short, (his entropic birth rate is high), more effort will be required to increase the target's average uncertainty an equivalent proportion.

V.2 Stability

The preceding paragraph dealt strictly with stationary or equilibrium behavior and analyzed the sensitivity of the equilibrium condition to variation of system parameters, in particular the counter-C^3 coefficients γ_{yx} & γ_{xy}. In this section, we wish to investigate the dynamic behavior of the system. We can describe rather thoroughly the system behavior of this non-linear system for small deviations from equilibrium using a technique from non-linear mechanics.[4]

The analysis begins by translating the equations to the stationary point by the change of variables

$$\begin{aligned} x' &= x - x_1 \\ y' &= y - y_1 \end{aligned} \qquad (23)$$

which leads to

$$\begin{aligned} \dot{x}' &= [(\alpha_2 y_1 - \alpha_1)x' - \alpha_2(1-x_1)y' + \alpha_2 x'y'] \quad Y_{max} \\ \dot{y}' &= [(\beta_2 x_1 - \beta_1)y' - \beta_2(1-y_1)x' + \beta_2 y'x'] \quad X_{max} \end{aligned} \qquad (24)$$

---

[4] See, for example; Minorsky, "Non-Linear Mechanics", J.W. Edwards, Ann Arbor, 1947.

For small displacements from equilibrium, the product terms may be neglected so that motion near the stationary point is described by the solution of the coupled linear dynamical equations,

$$\dot{x}' = [\alpha_2 y_1 - \alpha_1)x' - \alpha_2(-x_1)y'] \; Y_{max}$$
$$\dot{y}' = [(\beta_2 x_1 - \beta_1)y' - \beta_2(1-y_1)x'] \; X_{max} \qquad (25)$$

Before undertaking a general study of these equations, let us consider conditions analogous to those of Case 1 described in part V.1 above, i.e. one-way countering of Y on X such that $\beta_2=0$. For simplicity, we shall further prescribe that $X_{max} = Y_{max}$, and that $v_{sx} + v_{ix} = v_{sy} + v_{iy}$. With these restrictions we are led to solutions

$$y' = y'(0) \exp\left[-(V_{IY} + V_{SY})t\right]$$
$$x' = x'(0)\exp\left[-(V_{IX}+V_{SX}+\gamma_{yx}(1-x_1)X_{max})t\right] - y'\frac{(1-x_1)}{(1-y_1)}\left[1-\exp[-\gamma_{yx}(1-x_1)X_{max}t]\right] \qquad (26)$$

where $x'(0)$ and $y'(0)$ are the initial perturbations and we recall that V_{IY}, V_{SY}, V_{IX}, V_{SX} are the non-normalized birth and death rates for X & Y.

We see that Y, which is not being actively countered, has the same dynamic behavior as in Eq (3), simple exponential decay back to equilibrium with time constant $(V_{sy} + V_{iy})^{-1}$. However, perturbations of X's entropy are actually forced back to equilibrium more rapidly. Moreover, although perturbations in X do not affect Y, displacements of Y from equilibrium do cause variations in X, but always <u>in the opposite sense</u>. Thus, if Y has a temporary loss of knowledge, X will obtain a temporary increase, and vice versa (See Figure 4). Note that X's maximum good fortune (or bad as the case may be) will be delayed from the time of Y's maximum loss of knowledge. The time delay of the maximum is given by

$$\frac{t_D}{T_R} = \frac{\ln \dfrac{\gamma_{yx}(1-x_1)}{v_{ix} + v_{sx}}}{\gamma_{yx}(1-x_1)/(v_{ic}+v_{sx})} \qquad (27)$$

where $T_R = (V_{IX} + V_{SX})^{-1}$ is the uncoupled relaxation time of the system. Eq (27) is plotted in Figure 5. We see that for very small couplings, the maximum effect is delayed about one system time constant, whereas for large couplings, the maximum effect occurs almost immediately.

A more general analysis of Eq(s) (25) is made easier by considering the reversible linear transformation

$$\xi = A x' + By' \qquad \eta = Cx' + Dy' \tag{28}$$

with A,B,C,&D chosen such that

$$\dot{\xi} = S_1\xi \quad , \quad \xi = \xi_o e^{S_1 T} \qquad \dot{\eta} = S_2\eta \quad , \quad \eta = \eta_o e^{S_2 t} \tag{29}$$

and $\begin{vmatrix} A & B \\ C & D \end{vmatrix} \neq 0$. Eq's (29) are known as the cannonical form.

Following Minorsky[4], one finds that the exponential coefficients, $S_{1,2}$ are given by roots of the characteristic equation,

$$S^2 - p S + q = 0 \qquad S_{1,2} = p/2 \pm \frac{1}{2} (p^2-4q)^{\frac{1}{2}} \tag{30}$$

where in our case:

$$p = (\alpha_2 y_1 - \alpha_1)Y_{max} + (\beta_2 x_1 - \beta_1)X_{max}$$

$$q = \alpha_2\beta_2\left[\left(\frac{\alpha 1}{\alpha_2} - y_1\right)\left(\frac{\beta 1}{\beta_2} - x_1\right)-(1-x_1)(1-y_1)\right] X_{max}\, Y_{max} \tag{31}$$

$$p^2-4q = \left[(\alpha_2 y_1 - \alpha_1)Y_{max} - (\beta_2 x_1 - \beta_1)X_{max}\right]^2 + 4\alpha_2\beta_2(1-x_1)(1-y_1)X_{max}Y_{max}$$

The nature of the behavior of the cannonical equations, and hence x' and y' through the inverse linear transformation, is determined by the location in p,q space. Figure 6 shows the type of dynamical behavior that obtains near equilibrium in various regions of p,q space.

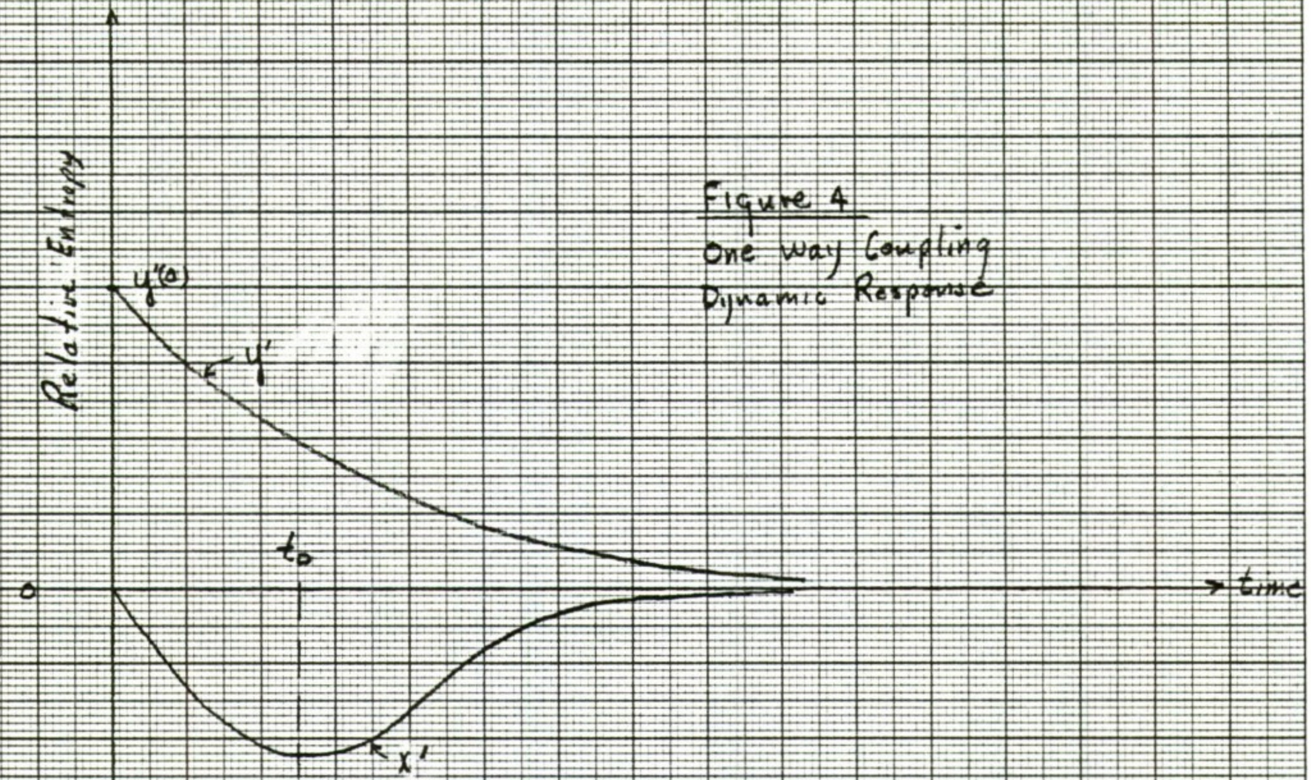


Figure 4
One way Coupling
Dynamic Response

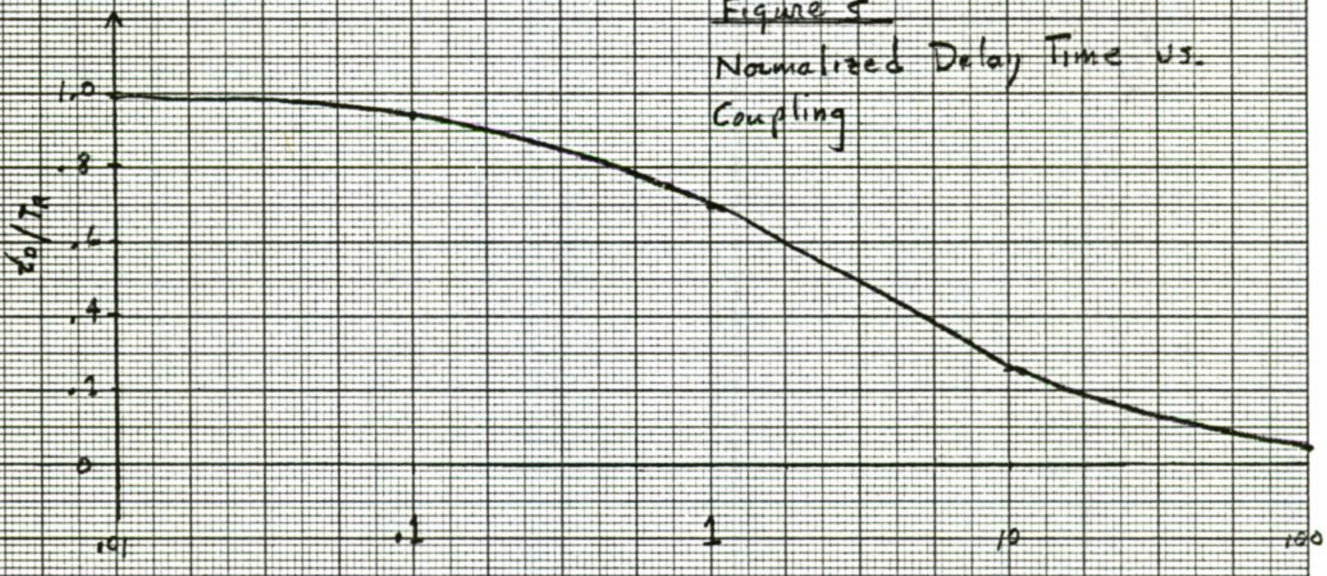


Figure 5
Normalized Delay Time vs. Coupling

Since $\alpha_1 > \alpha_2 y_1$ and $\beta_1 > \beta_2 x_1$, $p < 0$. It is also obvious that $\alpha_2\beta_2 > 0$ so that $p^2-4q > 0$ and we are never in Regions II or III of Figure 6. We can also see that since $\frac{\alpha_1}{\alpha_2} > 0$, and $\frac{\beta_1}{\beta_2} > 1$, that $q > 0$ since $\alpha_2\beta_2 > 0$.

Therefore, we see that p and q are always found in Region IV of Figure 6 and the equations exhibit stable nodal point behavior near equilibrium. That is to say, in Region IV, S_1 and S_2 are always real negative numbers, and therefore ξ and η are simple damped exponentials.

Our small perturbation solutions are then of the form (see Minorsky, p. 44)

$$x' = \left[S_2 + Y_{max}(\alpha_1 - \alpha_2 y_1)\right] \xi_o e^{S_1 t} + \left[S_1 + Y_{max}(\alpha_1 - \alpha_2 y_1)\right] \eta_o e^{S_2 t}$$
$$y' = X_{max} \beta_2(1-y_1) \xi_o e^{S_1 t} + \eta_o e^{S_2 t} \tag{32}$$

which are just linear combinations of the damped exponentials. Thus the system is returned to equilibrium with the time constants $-S_1^{-1}$ and $-S_2^{-1}$ determined according to Eq's (30). The constants ξ_o and η_o satisfy the initial conditions

$$x'(0) = (S_2 + \alpha_1 - \alpha_2 y_1)\xi_o + (S_1 + \alpha_1 - \alpha_2 y_1)\eta_o$$
$$y'(0) = \beta_2(1-y_1)(\xi_o + \eta_o) \tag{33}$$

Consider now the nature of these solutions for conditions somewhat analogous to Case 2 of part V.1, the "matched case". In particular, again let $X_{max} = Y_{max}$, $\gamma_{yx} = \gamma_{xy}$, $(\alpha_2 = \beta_2)$, $x_1 = y_1$ and $v_{sx} = v_{sy}$. (This assure that $\alpha_1 = \beta_1$). Under these "dynamically matched" system conditions we find that

$$p = 2(\alpha_2 x_1 - \alpha_1)X_{max}$$
$$(p^2-4q)^{\frac{1}{2}} = 2\alpha_2(1-x_1)X_{max} \tag{34}$$

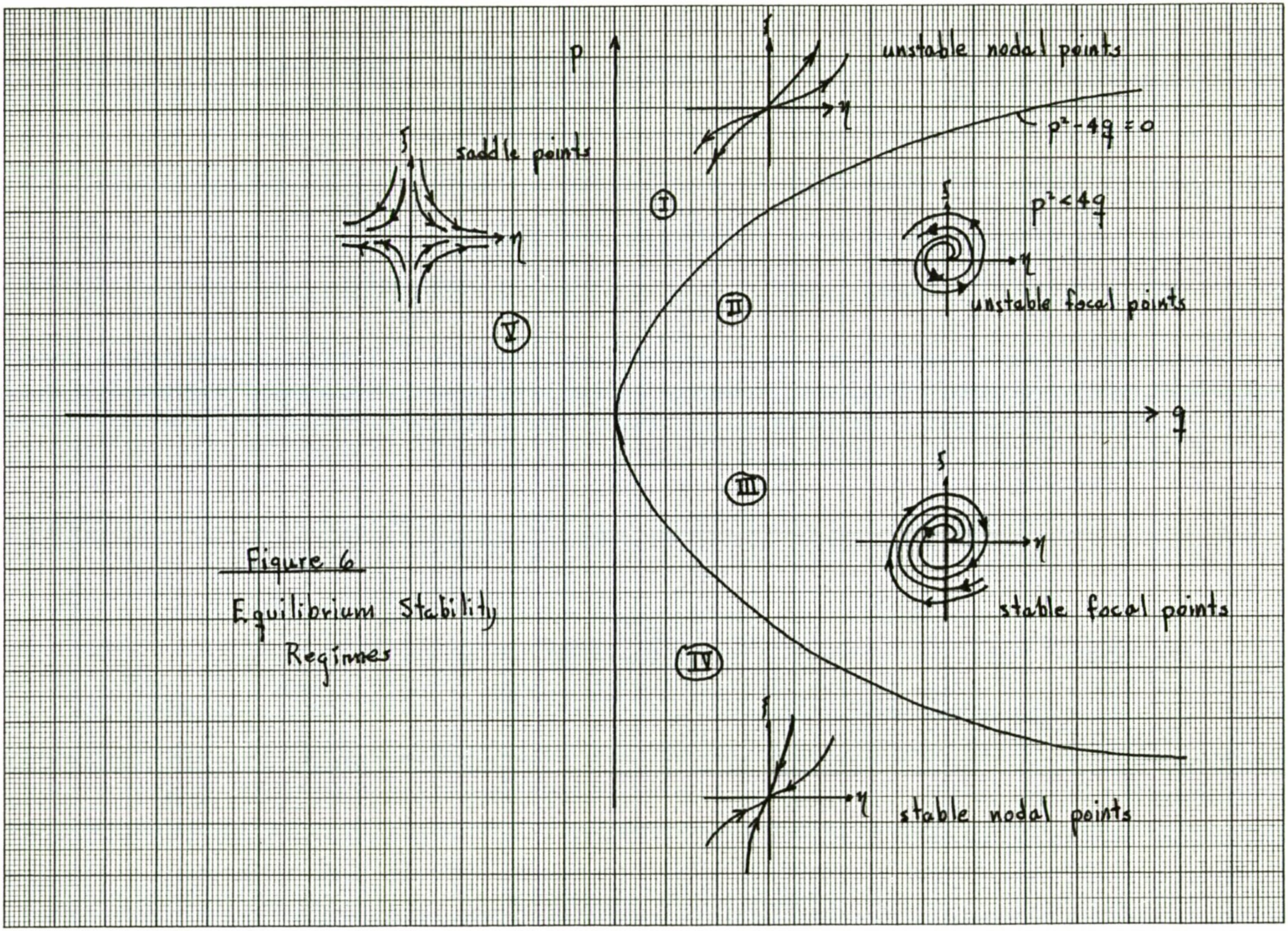


Figure 6
Equilibrium Stability Regimes

so that the characteristic roots of Eq (30) are simply

$$S_1 = -(\alpha_1-\alpha_2)X_{max} = -(V_{IX} + V_{SX})$$
$$S_2 = S_1 - 2\,\gamma_{yx}(1-x_1)X_{max} \tag{35}$$

The dynamic behavior (Eq's (32)) becomes

$$x' = (\Sigma)\ e^{S_2 t} + (\Delta)\ e^{S_1 t}$$
$$y' = (\Sigma)\ e^{S_2 t} - (\Delta)\ e^{S_1 t} \tag{36}$$

where, $\Sigma = \dfrac{x'(0) + y'(0)}{2}$, $\Delta = \dfrac{x'(0) - y'(0)}{2}$

Suppose the system is initially disturbed so that $\underline{x'(0) = -y'(0)}$.

Then Eq's (36) become

$$x' = x'(0)\ e^{-(V_{IX} + V_{SX})t}$$
$$y' = y'(0)\ e^{-(V_{IY} + V_{SY})t} \tag{37}$$

which are identical to a completely uncoupled system (See Eq (3)). However, suppose the system receives perturbations of the same signs, i.e. $\underline{x'(0) = y'(0)}$. Then

$$x' = x'(0)\ e^{-(V_{IX} + V_{SX} + 2\gamma_{yx}(1-x_1)X_{max})t}$$
$$y' = y'(0)\ e^{-(V_{IY} + V_{SY} + 2\gamma_{xy}(1-y_1)Y_{max})t} \tag{38}$$

and the systems are driven back toward equilibrium even faster than when no coupling exists by virtue of the counter-C^3 activities. Finally, suppose $\underline{x'(0) = x'(0) \text{ and } y'(0) = 0}$. Then

$$x' = \frac{x'(0)}{2}\ e^{-(V_{IX} + V_{IY})t}\left[1 + e^{-2\gamma_{yx}(1-x_1)X_{max}t}\right]$$
$$y' = -\frac{x'(0)}{2}\ e^{-(V_{IX} + V_{IY})t}\left[1 - e^{-2\gamma_{xy}(1-y_1)Y_{max}t}\right] \tag{39}$$

and Y will experience a change of entropy in the opposite sense of X. Again the maximum effect will be delayed as in the one way coupling case according to Eq (27), but with the effect of the counter-C^3 coefficient, γ_{xy}, doubled.

To summarize, the equilibrium points are always stable, and non-oscillatory for the counter-C^3 model investigated in this section. Small perturbations of the system from equilibrium are powerfully forced back to equilibrium with time constants equal to or less than those of the individual systems. However, counter-C^3 efforts do cause a perturbation in one system to appear as a perturbation in the other system, but with some time delay.

V.3 Dynamic Behavior Far From Equilibrium

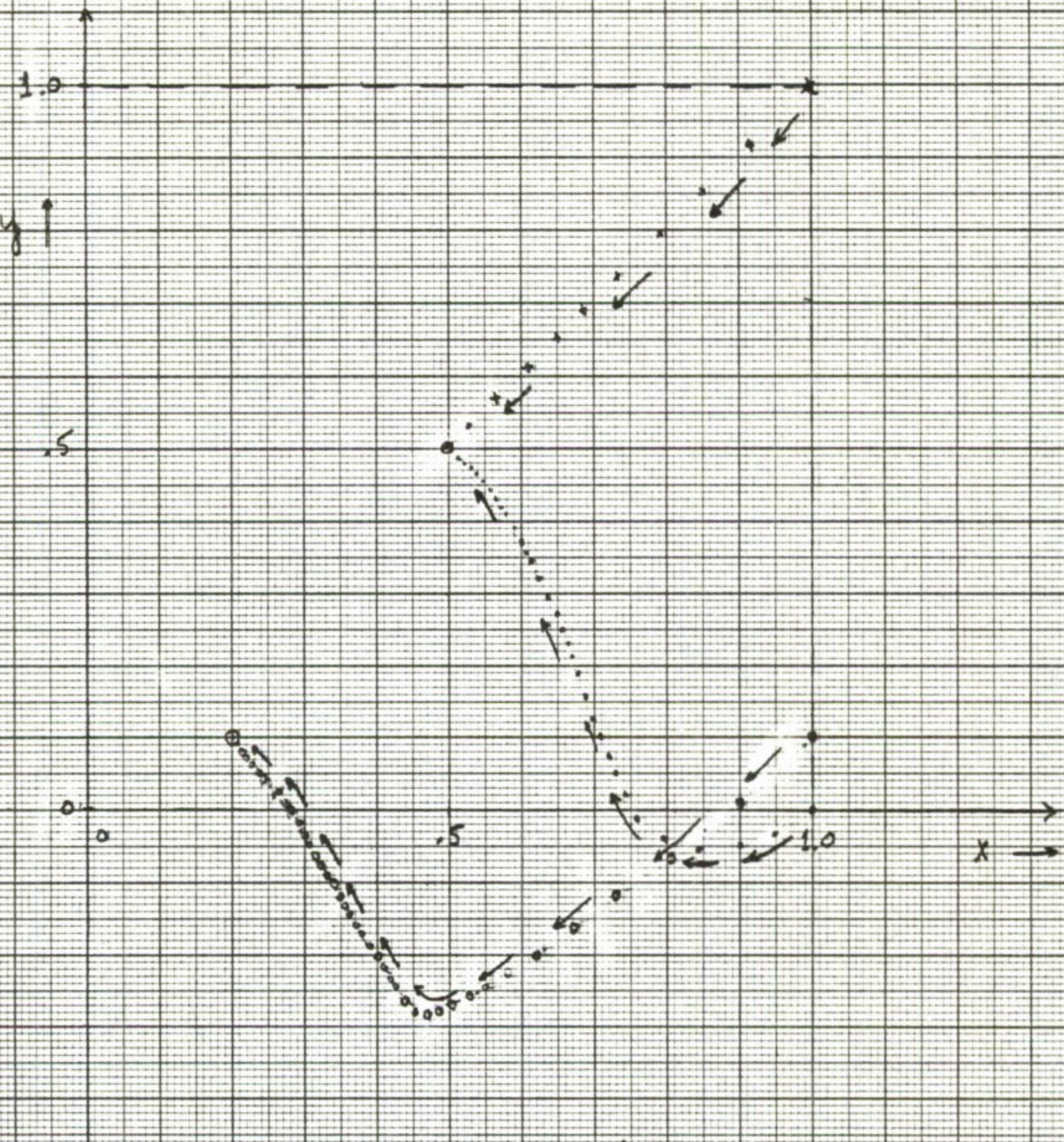
It is difficult to find general results for the behavior of the system far from equilibrium. However, a modest numerical investigation of Eq's (24) has revealed the following:

1) For initial perturbations of about 0.1 (i.e. one tenth the entire entropy range), the behavior seems well described by Eq's (32), the small perturbation result.

2) For large initial displacements, e.g. 0.9, the behavior is still stable nodal point in character and the entropies return to equilibrium smoothly. However, the "modeled entropy" for one system may temporarily go negative, which is physically impossible, although neither entropy ever appears to exceed its maximum value.

3) The rate of return to equilibrium seems proportional to the counter-C^3 coefficients. Strong counter-C^3 activities appear to strengthen the forces returning the system to equilibrium (Also see Eq's (39)).

We have shown some typical dynamic trajectories in Figure 7 for 2 equilibrium points and 3 initial conditions. Note the asymmetry of the



-.5

Figure 7
Typical
Dynamic Behaviour Far
From Equilibrium

$V_{Ix}+V_{Sx}=V_{Iy}+V_{Sy}=.02$

$\gamma_{rx}=\gamma_{xr}=.12$

(Equilibrium points and initial conditions apparent from plots)

trajectories returning to (.5,.5) from (1,0) and (1,1). Also the one returning from (1,1), total uncertainty is restored to equilibrium much more quickly than the one coming from total uncertainty of X and perfect knowledge of Y, (same signs versus opposite signs).

Two of the trajectories become negative although beginning and ending inside the physically realizable region. What will happen if a non-negativity constraint is imposed is not known, but could rather easily be investigated numerically. What does seem to be true is that even starting on a boundary, the entropies do not blow up or trend toward the other equilibrium point in the first quadrant but outside the physical region. One should be cautioned that this conclusion is based on a limited numerical investigation of the non-linear dynamical system equations and further investigation could indeed prove otherwise.

VI. Discussion

The model we have proposed for C^3 information dynamics based on entropy includes a non-linear term to account for counter-C^3 activities of two sides engaged in an information war. Before we discuss the meaning and implication of our analytical results, let us briefly recount some other applications of this class of equations.

VI.1 Population Dynamics

An interesting example is provided by the Lotka-Volterra model of population dynamics, originally devised to explain temporal oscillations in the occurrence of fish in the Adriatic Sea. If we let the prey fish be x and the predator fish be y, then the model is described by,

$$\begin{aligned} \dot{x} &= \alpha_1 x - \alpha_2 xy \\ \dot{y} &= -\beta_1 y + \beta_2 xy \end{aligned} \tag{40}$$

where α_1 and β_1 are the prey birth and predator death rates respectively and α_2 and β_2 describe the prey losses and predator gains due to predator feeding. These equations have been studied extensively (along with more elegant versions) for describing population dynamics of conflicting species.[5] They have non-zero equilibrium points about which under some conditions they exhibit stable focal point behavior, unlike our counter-C^3 equations. Note that the non-linear terms are of opposite sign. As we shall see shortly, this is a requirement for focal point behavior.

VI.2 Models of Combat

Another example is provided by the Lanchester equations for armed combat.[6] Here we take x and y to be the sizes of the forces of two sides. Then

$$\begin{aligned} \dot{x} &= -\alpha_1 y - \alpha_2 xy \\ \dot{y} &= -\beta_1 x - \beta_2 xy \end{aligned} \tag{41}$$

include both terms most commonly considered. When $\alpha_2 = \beta_2 = 0$, Eq's (41) are of the form Lanchester termed "aimed fire" or "modern warfare". When $\alpha_1 = \beta_1 = 0$, Eq's (41) are of the form he called area fire or "ancient warfare". Although a non-zero equilibrium point exists in the general case of Eq's (41), we see that no non-zero equilibrium is possible for either of the special cases. However, an "exchange ratio", $\frac{\Delta x}{\Delta y}$, is obtained by dividing the two equations for these two cases. The solutions show the trajectories that x and y must follow as the forces are depleted. In particular, for "aimed" fire, the exchange ratio is

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[5] May, R.M., "Stability and Complexity in Model Ecosystems", Princeton Univ. Press, 1973.

[6] Taylor, J.G., "A Tutorial on Lanchester-Type Models of Warfare", Proc of 35th MORS Symposium, July 1975.

$$\frac{dx}{dy} = \frac{\alpha_1}{\beta_1} \quad \frac{x}{y} \tag{42}$$

$$\beta_1(x_i^2 - x^2) = \alpha_1(y_i^2 - y^2)$$

and for "area" fire

$$\frac{dx}{dy} = \frac{\alpha_2}{\beta_2}$$

$$\beta_2(x_i - x) = \alpha_2(y_i - y) \tag{43}$$

These results have been interpreted to show the advantage in concentrating forces to minimize losses when "aimed" fire prevails whereas concentrating forces in "area" fire is of no particular advantage. To illustrate, let $\alpha_2 = \beta_2 = \alpha_1 = \beta_1 = 1$ and let $y_f = 0$. Then for "area" fire $\frac{x_i - x_f}{y_i} = 1$ regardless of x_i so $X^{'s}$ and $Y^{'s}$ losses are matched. However, for "aimed" fire

$$\frac{x_i - x_f}{y_i} = \frac{x_i}{y_i}\left[1 - \left(1 - \frac{y_i^2}{x_i^2}\right)^{1/2}\right) \tag{44}$$

which for $\frac{y_i}{x_i} << 1$ is approximated by

$$\frac{x_i - x_f}{y_i} = \frac{1}{2}\,\frac{y_i}{x_i} \tag{45}$$

For example, if $x_i = y_i$ then of course the losses are matched (Eq. (44)), however, if X initially concentrates all his forces against Y, so, say $x_i = 4y_i$, then $X^{'s}$ losses are only 1/8 of $Y^{'s}$ losses.

One of the characteristics of Lanchester's specialized equations are that their dynamical solutions, like our C^3 equations, are sums of real exponentials. However, since they have no non-zero stable point, their equilibrium behavior is not of great interest. Also, the solutions can go negative, which is not physically consistent with reality.

VI.3 Alternative C^3 Information Models

Consider the pair of equations

$$\begin{aligned} \dot{x} &= \alpha_0 - \alpha_1 x + f(x,y) \\ \dot{y} &= \beta_0 - \beta_1 y + g(y,x) \end{aligned} \tag{46}$$

Let us list some of the other options for $f(x,y)$ and $g(y,x)$, along with the one analyzed in Part V, and their interpretations.

Alternative	$f(x,y)$	$g(y,x)$	Entropy Birth (Death) Rate
a.)	$\alpha_2(1-y)$	$\beta_2(1-x)$	Counter-C^3 efforts create entropy in proportion to own knowledge.
b.)	$\alpha_2(1-x)(1-y)$	$\beta_2(1-y)(1-x)$	Counter-C^3 efforts create entropy in proportion to product of both sides knowledge.
c.)	$\alpha_2 x(1-y)$	$\beta_2 y(1-x)$	Counter-C^3 efforts create entropy in proportion to self knowledge and opposition's ignorance.
d.)	$-\alpha_2 y(1-x)$	$-\beta_2 x(1-y)$	Intelligence efforts destroy entropy in proportion to self knowledge and opposition's ignorance.

Alternatives a.), b.) and c.) are identical to the counter-C^3 coupling options a.), b.) and c.) of Part IV. Alternative b.) is the case analyzed extensively in Part V. Alternative d.) listed here is a new concept. It proposes that instead of counter-C^3 activities to increase the opponent's entropy, the players focus on special intelligence activities to reduce their own uncertainty and that the entropy death rate by such means will be proportional to one's own knowledge and the opposition's ignorance.

The implications of these four alternatives for dynamic stability are listed below:

Alternative	$Re[S_1 \& S_2]$	$Im[S_1 \& S_2]$
a.)	Both < 0 (stable)	0 (no focal points)
b.)	Both < 0 (stable)	0 (no focal points)
c.)	Either or both may be > 0 (may be unstable)	0 (no focal points)
d.)	Either or both may be > 0 (may be unstable)	0 (no focal points)

We see that models c.) & d.) may lead to unstable behavior near equilibrium points that would cause one or both entropies to blow-up. However, no model produces focal point behavior near equilibirum. This is because in all models, $\alpha_2\beta_2 > 0$, which forces the imaginary parts of the roots of the characteristic equation to be zero. (This is not so with the Lotka-Volterra equations for population dynamics.)

An interesting avenue for further research will be to investigate combinations of counter-C^3 and intelligence models both for stable and unstable as well as focal point behavior and for equilibrium sensitivity to intelligence vice counter-C^3 efforts.

VII. Summary

In this paper we have proposed a model for C^3 information dynamics incorporating the effects of counter-C^3 activities. The model is based on the inevitable growth of uncertainty inherent in military situations and the concept of information sensors and sources acting as constraints that maintain uncertainty below its worst possible value. System entropy dynamics of two opposing sides are characterized by natural "birth" and "death" rates of entropy. Counter-C^3 activities are introduced as additional growth terms that depend in some way on the entropy of one or both players.

The dependence analyzed extensively in this paper in Part V models counter-C^3 effectiveness as being in direct proportion to the product of the two system's knowledges, $(X_{max}-X)\ (Y_{max}-Y)$, where X and Y are the two entropies and X_{max} and Y_{max} the largest amount of uncertainty (or information) possible in the two systems respectively. It is shown that for this kind of coupling, the relative shift of system equilibrium is directly proportional to the ratio of the coupling coefficient to the system's natural entropy birth rate. Furtheremore, it is shown that small perturbations from equilibrium are restored to equilibrium by the system forces, i.e. the system is ultra-stable, but that 1) perturbations of X and Y with the same sign are restroed much more rapidly than perturbations of opposite sign, and 2) a perturbation in one system induces a delayed perturbation in the other system of the opposite sign. Thus, if X becomes fortuitously more knowledgeable by chance, Y will in turn some time later become more ignorant and vice versa. It is also shown the system's dynamical trajectories near equilibrium are described by sums of exponentials with real coefficients. Such equilibrium points are called "nodal points". (In contrast, "focal point" systems have exponentials with complex coefficients

in which the trajectories spiral into the equilibrium point. See Figure 6.)

A modest investigation of the dynamics far from equilibrium show the system always returns to equilibrium; but it is possible for the "modeled entropy" of one or both systems to temporarily become negative, a physically impossible condition for the "real entropy". The importance of a non-negativity constraint on system behavior far from equilibrium requires further investigation.

Finally, after a brief review of two other well known applications of coupled non-linear state equations, the Lanchester combat equations and the prey-predator equations, several alternative counter-C^3 models and an "intelligence" model are proposed. It is shown that these models too exhibit nodal point behavior, unlike for example, the predator-prey model, which may exhibit focal point behavior. However, the intelligence model and a counter-C^3 model where the entropy rates depend on the product of one's own knowledge and the opposition's ignorance, are not necessarily stable. That is, small deviations from equilibrium may cause one, or the other, or both entropies to diverge.

It is clear that further theoretical investigation of entropy models is needed, along with some modest simulations of actual systems, to determine the utility of this approach for the evaluation of specific C^3I alternatives.

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